From numbers to strings and branes

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Based on papers

From Matrix Models and quantum fields to Hurwitz space and the absolute Galois group
arXiv:1002.1634[hep-th] ; Robert de Mello Koch, Sanjaye Ramgoolam
Toric CFTs, Permutation Triples, and Belyi Pairs
http://arxiv.org/abs/1012.2351; Vishnu Jejjala, Sanjaye Ramgoolam, Diego Rodriguez-Gomez
“Holomorphic maps and the complete 1/N expansion of 2D SU(N) Yang-Mills,”
http://arxiv.org/abs/0802.3662 ; Yusuke Kimura, Sanjaye Ramgoolam

Introduction

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$p(n)$ counts

- States in a free 1+1 dimensional boson Fock space.
- A free 1+1 dimensional fermion Fock space
- Multi-traces of 1-matrix
- Conjugacy classes of S_n ; cycle structures of permutations of n objects.
- Young diagrams with n boxes.

All the entries are related.

There are mechanisms/techniques of physical relevance to relate them.

Bosonization, Van der Monde Determinants, group theory of permutation groups.

Asymptotics of $p(n)$ are responsible for Hagedorn transition.

Seems a safe statement : that we understand the physics of $p(n)$.

In this talk we will consider some other identities between sequences, in some way related to $p(n)$, which involve

on the one hand, geometry of holomorphic maps from a string worldsheet to space-time

On the other hand, the all orders $1/N$ expansion of Matrix integrals.

The identities are understood using the idea of gauge-string duality.

Unlike boson-fermion correspondence, and other dualities, say T-duality, gauge-string duality is very much work in progress.

We know many examples, we don't know how to derive it, and we don't really know what makes it work.

So finding the identities between such sequences is a good guide to uncovering new aspects of gauge-string dualities.

The sequences at hand involve the $1/N$ expansion of the complete set of gauge-invariant correlators in a Hermitian matrix model.

$$\int dX \;\; e^{-\frac{1}{2}trX^2}\mathcal{O}(X)$$

The observables are general traces :

$$\begin{array}{l} trX \\ trX^2, trXtrX \\ trX^3, trX^2trX, trXtrXtrX \end{array}$$

For a fixed number n of X's we have $p(n)$ observables.

This simple model is of interest in context of AdS/CFT in a special limit which is far from semi-classical gravity.

See Gopakumar (arXiv:1104.2386) ”What is the Simplest Gauge-String Duality?” for further explanation of motivations to study this model.

Take the canonical gauge-string duality of $N = 4$ SYM with $AdS_5 \times S^5$.

One of the most remarkable features is the emergence of geometry. There is an explicit $R^{3,1}$ as base space of the gauge theory. No sign of an S^5.

Consider the theory in radial quantization. Put it on $S^3 \times R$ and go to zero coupling. The scalars get a mass due to conformal coupling.

Restrict to zero angular momentum for the S^3 (s-wave reduction). Focus on just one hermitian scalar and further drop all time dependence (dimensionally reduce on R)

This rather brutal reduction yields the hermitian matrix model :

Has a gauge-string dual !

And shows emergence of geometry, a two-sphere $S^2 = \mathbb{P}^1$ that wasn't there to begin with !!

Holomorphic maps between Riemann surfaces are very nice maps

Locally they always look like

$$w = z^n$$

Globally they are branched covers.

All world-sheets involved are non-singular. Branching just means that the derivative of the map vanishes.

Identifying the combinatoric sequences that arise from the $1/N$ expansion of the Hermitian matrix model involves some techniques for manipulating operators in tensor space.

Oparators : Matrices or permutations –both of which act on tensor products of a fundamental vector space (of dimension N)

The operators in question start off being matrices which morph into (sums of) permutations after we do Wick contractions.

These basic tensor space manipulations connect the combinatorics of the $1/N$ expansion directly to the geometry of holomorphic maps – using Riemann existence theorem

In addition to setting up the math foundation of GR, he also set up the foundations of the simplest gauge-string duality !!

We focus first on the case of hermitian matrix model.

A special class of holomorphic maps appear : three branch points. They are important in number theory, and are called Belyi maps, in honour of Belyi – who understood the number theory meaning (80's)

Very similar results hold for the sector of half-BPS correlators ; 2-dimensional Yang-Mills theory, multi-matrix models, higher dimensional free hermitian matrix field theory.

The connection : Large N combinatorics - Holomorphic maps is very generic.

Reasonable to expect Holomorphic maps between Riemann surfaces will be key to understanding the string dual of free (zero coupling (and weak coupling?)) yang Mills theory – which is far from the semiclassical gravity limit of AdS_5 with large radius.

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Lesson : We don't fully understand all the secrets of $p(n)$.

OUTLINE

- The Matrix model : From correlators to counting triples of permutations and holomorphic maps with three branch points.

- Matrix model Feynman graphs and Dessins.
- Physical Interpretation : Target space of $\mathbb{P}^1$; holo map counting results from 90's semiclassics of matrix models. Topological strings, Localization to holomorphic maps ; Open problems. (see Gopakumar-1104 for some new identities between top A-model $\mathbb{P}^1$ string correlators and hermtian matrix correlators, at genus zero.)

OUTLINE

- Why 3 ? Curves over Algebraic numbers and Belyi. *Gal*$(\bar{\mathbb{Q}}/\mathbb{Q})$ organizes the Feynman graphs into orbits : with *Aut f* an invt.

- Other Matrix correlators related to permutations and holo maps : Half-BPS sector of complex matrix model (Tom Brown, http://arxiv.org/abs/1009.0674)
  Normal-ordered correlators in Hermitian matrix model,
  Multi-matrix models,
  Free matrix field theory in higher dimensions
  2d YM string – recent holo results for non-chiral YM2 (kimura-Ramgoolam: 0802)

- Classifying 3-brane gauge theories at CY singularities with “dimer models” and Permutaions triples and Belyi maps.